Sketch the graph of the following and determine if it is a domain:

1. \( \text{Im} \left( \frac{-z}{z} \right) = -2 \)
2. \( \text{Im}(z - i) = \text{Re}(z + 4 - 3i) \)
3. \( |z + 2 + 2i| = 2 \)
4. \( |\text{Re}(z)| > 2 \)
5. \( \text{Im}(z - i) < 5 \)
6. \( \text{Re}(z^2) > 0 \)
7. \( \text{Im}(z - i) = \text{Re}(z + 4 - 3i) \)
8. \( 0 \leq \text{Arg}(z) \leq \frac{2\pi}{3} \)
9. \( |z - i| > 1 \)
10. \( 2 < |z - i| < 3 \)

For #11 - 13, Describe the set of points that satisfy the statement. Description includes

a) an equation or inequality of \( x \) and \( y \),
b) a sketch
c) name the type of the shape (i.e., straight line, ellipse, parabola, etc.).

11. \( |z + 1| = |z - i| \)
12. \( |\text{Re}(z)| < |z| \)
13. \( z^2 + (\bar{z})^2 = 2 \)
Homework # 2 Due Monday 2/1/2016

14) Make a detailed plot of the following set:
   \[ |\text{Arg}(z)| \leq \frac{\pi}{3} \text{ and } \text{Re}(z) \leq 1 \]
   Is the set a domain? Explain.

15) Make a detailed plot of the following set:
   \[ \text{Arg}(\sqrt[3]{1}) - 30^\circ < \text{Arg}(z) < \text{Arg}(\sqrt[3]{1}) + 30^\circ \text{ and } 1 < |z| < 2 \]
   Is the set a domain? Explain.
Homework # 2  Due Monday 2/1/2016

#1
\[ I_m(\overline{z}) = -2 \]
\[ I_m(-x + iy) = -2 \]
\[ I_m(-x - iy) = -2 \]
\[ I_m(-x + iy) = -2 \]
\[ y = -2 \]

Not a domain (not open)

#2
\[ I_m(z - i) = Re(z + y - 3i), \ z = x + iy \]
\[ I_m(x + iy - i) = Re(x + iy + y - 3i) \]
\[ I_m(x + i(y - i)) = Re(x + y + i(y - 3)) \]
\[ y - 1 = x + y \]
\[ y = x + 5 \]

Not a domain (not open)

#3
\[ |z + 2 + 2i| = 2, \ \Rightarrow \ circle, \ center = -2 - 2i \]
\[ I_m \]
\[ Not \ a \ domain \ (not \ open) \]
Homework # 2 Due Monday 2/1/2016

#4
\[ |Re(z)| > 2 \]
\[ |x| > 2 \]
\[ x > 2 \quad \text{or} \quad -x > 2 \]
\[ \Rightarrow x < -2 \]
Boundaries @ \( x = 2 \) & \( x = -2 \)
Not a domain (not connected)

#5
\[ \text{Im}(z-i) < 5 \]
\[ \text{Im}(x+iy-i) < 5 \]
\[ \text{Im}(x+i(y-1)) < 5 \]
\[ y-1 < 5 \]
\[ y < 6 \]
It is a domain
#6 \[ Re(z^2) > 0 \]
\[ Re((x+iy)^2) > 0 \]
\[ Re(x^2 + 2xiy - y^2) > 0 \]
\[ x^2 - y^2 > 0 \]
\[ x^2 > y^2 \quad (x \neq 0 \text{ by inspection}) \]

For \( x > 0 \)
\[ x^2 > y^2 \Rightarrow x > +y \]

For \( x < 0 \)
\[ x^2 > y^2 \Rightarrow x < +y \]

Combine:

Not a domain
(not connected)
Homework # 2  Due Monday 2/1/2016

#7  \[ \text{Im}(z-i) = \text{Re}(z+i-3i) \]
    \[ \rightarrow \text{SAME AS } #2 \]

#8  \[ 0 \leq \text{Arg}(z) \leq \frac{2\pi}{3} \], substitute \( z = re^{i\theta} \)

\[ \text{Arg}(re^{i\theta}) = \theta \]

\[ 0 \leq \theta \leq \frac{2\pi}{3} \]
\[ \text{NOT A DOMAIN} \]
\[ \text{(NOT OPEN)} \]
\[ \text{(NOTE ALSO: } z = 0 \text{ IS NOT)} \]
\[ \text{PART OF THE SET} \]

#9  \[ |z-i| > 1 \]

CIRCLE
CENTER = +i
RADIUS = 1

#10  \[ 2 \leq |z-i| \leq 3 \]

ANNULUS
CENTER = +i
INNER RADIUS = 2
OUTER RADIUS = 3

IS A
DOMAIN
Homework # 2  Due Monday 2/1/2016

# 11
\[ |z + 1| = |z - i| \]
\[ |x + iy + 1| = |z + iy - i| \]
\[ |(x+1) + i(y)| = |(x) + i(y-1)| \]
\[ \sqrt{(x+1)^2 + y^2} = \sqrt{(x)^2 + (y-1)^2} \]
\[ (x+1)^2 + y^2 = x^2 + (y-1)^2 \]
\[ x^2 + 2x + 1 + y^2 = x^2 + y^2 - 2y + 1 \]

# 12
\[ |\text{Re}(z)| < |z| \]
\[ |x| < |x + iy| \]
\[ \pm x < \sqrt{x^2 + y^2} \]
\[ x^2 < x^2 + y^2 \]
\[ 0 < y^2 \]
\[ 0 < \pm y \]
\[ y \neq 0 \]

boundary is a straight line
#13
\[ z^2 + (\bar{z})^2 = 2 \]

Substitute \( z = x + iy \)

\[ (x + iy)^2 + (x - iy)^2 = 2 \]
\[ x^2 + 2ixy - y^2 + x^2 - 2ixy - y^2 = 2 \]
\[ 2x^2 - 2y^2 = 2 \]
\[ x^2 - y^2 = 1 \]

Hyperbola with horizontal transverse axis
Asymptotes at \( y = x \) and \( y = -x \)
Distance from origin to center = \( \sqrt{2} \)

Asymptotes
Homework # 2  Due Monday 2/1/2016

14) \[ |\text{Arg}(z)| \leq \frac{\pi}{3} \text{ and } \text{Re}(z) \leq 1 \]

Angle of \( z \) is between \( \pm \frac{\pi}{3} \ (\pm 60^\circ) \)

Not a domain

(Not open)
Homework # 2  Due Monday 2/1/2016

\[ 3 \sqrt{1} = 1 \angle 0^\circ, \quad 1 \angle 120^\circ, \quad 1 \angle -120^\circ \]

\[
\begin{align*}
\text{Arg}(e^{j\pi/3}) &< \text{Arg}(z) < \text{Arg}(e^{j\pi/3}) + 30^\circ \\
\text{Arg}(z) &\in \left\{ 0^\circ, 120^\circ, -120^\circ \right\} \\
1 < |z| < 2
\end{align*}
\]

The set is the intersection ("and") of these two regions.

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The set is comprised of the shaded areas.

This is not a domain (not connected).