SHOW ALL WORK!

(1) Consider $f(z) = 3(z - i)^3 + (-1 + 10i)(z - i)^2 + (-11 + 3i)(z - i) - 2 - 5i$.
   $f(z)$ is a third order polynomial centered on $z_0 = i$.
   a) Find the re-centered polynomial centered on $z_1 = 0$, that is, find the Maclaurin series for $f(z)$.
   b) Now find the Taylor series for $f(z)$ centered on $z_2 = (1 - i)$.

(2) Consider $f(z) = \frac{1}{1-z} = 1 + z + z^2 + z^3 + z^4 + \ldots$.
   This is a Taylor series centered on $z_0 = 0$.
   a) What is the radius of convergence of this series?
   b) Now find $f(z)$ as a Taylor series centered on $z_1 = 2i$ expressed in sigma notation.
   c) What is the radius of convergence of the new series?

(3) Consider $f(z) = \sin z$. ($z$ is in radians!)
   a) Find the first four terms of $f(z)$ as a Taylor series centered on $z_0 = 8$.
      Find the numerical value of the coefficients to five significant figures.
      You may use your calculator to find $\cos(8)$ and $\sin(8)$ (you need both).
   b) Use your truncated Taylor series to estimate the value of $\sin(8.1)$ by finding the sum of the first four terms evaluated at $z = 8.1$.
   c) Compare that sum to the actual value of $\sin(8.1)$.
   d) (Extra) How many terms of the usual series (centered on the origin) would you have to compute to get the same degree of precision?

(4) Consider $f(z) = \sum_{n=0}^{\infty} [z(a + bi) + c + di]^n$ where $a, b, c, d \in \mathbb{R}$, $a + bi \neq 0$
   This is power series that is not in the standard arrangement.
   a) Rearrange the expression so that it is in proper form for a power series.
   b) What is the center of the series?
   c) What is the region of convergence?